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OSCILLATIONS AND WAVE MOTION OF A LIQUID
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THERMALLY DRIVEN OSCILLATIONS AND WAVE MOTION OF A LIQUID DROP

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Abstract

in the state of Leidenfrost boiling, liquid drops are observed to vibrate in a variety of modal patterns. Theories are presented which predict the frequency of oscillation and show that the observed modal patterns of drops correspond to the minimum energy oscillatory excitation state. High-speed photographic techniques were used to record these motions and substantiate the theories. An incipient temperature was also found for water drops in film boiling below which free oscillations do not exist. In addition to these oscillations, photographic sequences are presented which show that wave motion can exist along the circumference of the drop. Following the study of free oscillations, the system was mounted on a shaker table and the drop subjected to a range of forced frequencies and accelerations.

NOMENCLATURE

a	equilibrium radius of drop
a_n	amplitude coefficient
a^*	dimensionless, a , see table I
C_f	drop height correction factor, see eq. (14)
Γ_p	potential energy of drop
e	phase factor
f	frequency
g	local acceleration
g_c	gravitational constant
h	equilibrium drop height
h^*	dimensionless drop height, see table I
k	axial wave number
L	characteristic length, see table I
n	circumferential wave number
ΔP	pressure difference across interface
p	frequency
R_1	drop radius of curvature
R_2	drop radius of curvature
r	radial position of surface
t	time
V	drop volume
V^*	dimensionless drop volume, see table I
z	coordinate
α	angle cut on test plate, see Fig. 3
θ	angle, see Fig. 5
λ_c	critical wave length
λ_R	circumferential wave length
λ_z	axial wave length
ρ_L	liquid density
ρ_v	vapor density
σ	surface tension

INTRODUCTION

In 1756, Leidenfrost (Ref. 1) a German doctor of medicine placed a small drop of water on a red hot motionless spoon. He peered at a quiet clear crystalline drop which floated in the spoon. The drop appeared like a bright shiny diamond. For larger volumes of water on a flat hot surface, these drops (Refs. 2 and 3) can undergo vibrational oscillations of striking beauty. In such a manner, the free and forced oscillations of a deformable body can be studied inexpensively and conveniently.

Holter and Glasscock (Ref. 2) first presented photographic evidence that large drops in Leidenfrost boiling can undergo large amplitude vibrations similar to the variety of modal patterns shown in Fig. 1. One complete vibratory period of the four mode pattern is shown in Fig. 2. As seen in these photographs, drops can under distinct vibrations in a single mode.

Holter and Glasscock (Ref. 2) were unable to analytically correlate the natural frequencies of the vibration using the spherical drop theories presented in Refs. 4 and 5. Unaware of Ref. 2, Schoessow and Baumeister (Ref. 3) rediscovered the phenomena. They were able to correlate the natural frequencies of the drop by extending the spherical theory of Rayleigh (Ref. 4) to include cylindrical coordinates (pancake shaped drops). Interest in the effect of drop vibrations on the combustion process, for example, has stimulated continuing analytical research into the vibration and deformation of the liquid drops (Refs. 6 and 7). This phenomenon may also be of interest to the heat transfer specialist who is concerned with the fundamentals of the boiling process, or the theoretical physicist who sometimes needs a physical model to explain complex interactions of the nucleus, such as the popular water drop model of Bohr and Wheeler used in explaining nuclear fission. These modes of oscillation might also be used as an analog to help in the understanding of vibrations in structures, as well as a visual model of acoustic sources.

A model for the natural frequencies for a given modal pattern were presented in Ref. 3; however, it did not consider the problem of what modal patterns would exist for a given size drop, or why a critical temperature exists below which free vibrations for water no longer occur. The present paper addresses

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these problems and for the first time presents photographic sequences which show that wave motion can exist along the circumference of the drop. In addition to the study of free oscillations, the system was mounted on a shaker table and the drop subjected to a range of forced frequencies and accelerations. Free oscillations will be considered first.

FREE OSCILLATION

Free oscillations are said to occur when a drop vibrates in Leidenfrost film boiling on a fixed hot plate. This is in contrast to an experiment to be performed later in which a hot plate is mounted on a shaker table and forced to vibrate at frequencies near the natural frequencies of the drop.

Apparatus. Fig. 3 is a schematic diagram of the free oscillation test apparatus. The apparatus is a hot plate on which drops can undergo film boiling. When a drop of water is placed on the hot plate at temperatures in excess of 260°C , the vapor bubbles generated at the plate are of sufficient number that they coalesce and insulate the drop from the hot surface. Then, the drop can float quietly on a cushion of its own vapor. This is called Leidenfrost film boiling. Film boiling is an extremely poor means of removing heat; therefore, the life time of the drop on the plate is relatively long, from 1 to 10 minutes depending on the initial volume of the drop, plate temperature and fluid.

The vapor layer between the drop and the surface reduces the frictional drag on the drop and allows it to freely vibrate in patterns such as in Figs. 1 and 2. To visualize these patterns, a high speed motion picture camera was used to determine the frequency of oscillation and radius of the drop at any time.

Various plates ranging from 8 to 15 cm in diameter and various materials (aluminum, stainless steel, brass) were used in the study. The observed phenomena were generally found to be independent of these variables. A slight dishing of the plate (angle of 1° to 5°) was used to prevent the drop from floating across the surface.

In general, the number of nodes formed by a drop increases with the radius of the drop; that is, the eight node drop shown in Fig. 1 is considerably larger than the drop with four nodes. At large drop volumes, the vapor pressure beneath the drop increases in magnitude so that the vapor breaks through the center of the drop. The bubble break through process limits the number of nodes for water depending on the plate angle α . As part of this study, a plate was cut with an angle of 5° to increase the thickness of the water drop at its center. In experiments on this hot plate, drops with 9 and 10 nodes could be obtained as shown in Fig. 4. Larger volumes produced bubble break through; consequently, 10 nodes was the maximum observed.

In performing this experiment, a large drop of liquid was placed on the hot plate which was at sufficient temperature to produce Leidenfrost boiling. The drop usually took on a glittering ellipsoidal shape. After a short time the drop surface often became wrinkled and then suddenly jumped into nodal patterns. When this occurred, the camera was started to record the vibrations. At any instant, the drop might suddenly cease to vibrate and become

quiescent. Sometime later, after the drop volume has decreased due to vaporization, it may begin vibrating again but with a different number of nodes.

The drops normally oscillate between 10 and 30 hertz; consequently, the eye will not see clear and distinct modes as shown on the photographs. Rather, because of the persistence of vision in this frequency range, the drop will appear to have twice the actual modes shown in Figs. 1 or 4. Consequently, the odd modes, such as 3, 5, and 7 will not be seen by eye. Only an even number of nodes will be seen.

The radius of the drop at any time can be found from the movie sequences. This is the key variable in the correlations presented in the theory section. The drop volume associated with the radius could be found by knowing its initial volume and measuring the elapsed time of the drop on the plate. The volume could also be estimated from the radius by using a solution to the Laplace capillary equation

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{\Delta P}{\sigma} \quad (1)$$

The solution of eq. (1) was performed in Ref. 8 in which the numerical solutions to eq. (1) were approximated by the analytical solutions given in table I. The analytical results apply strictly for $\alpha = 0$; however, they were assumed to apply to α up to 5° .

Modal States. It was observed that the number of nodes for which a drop would vibrate generally increased with the surface area of the drop; however, this was not always true. A number of ideas were explored to explain the dependency of the drop nodes to the radius of the drop. The approach chosen was that of finding a functional which could be either maximized or minimized to yield the dependency of the mode on drop radius. Although this approach must be considered an art, it is a common and powerful technique used in many areas of engineering (Ref. 9). For example, in the theorem of least work in solid mechanics (Ref. 10, p. 212), the stress distribution in a body will be that which produces the least elastic energy. Using this basic idea from solid mechanics, an analysis is now presented which looks for the vibrational state of a liquid drop which has a minimum potential energy.

To determine the mode in which a liquid drop vibrates, a method of analysis is now presented which considers both the radial and axial curvature of a liquid drop in some modal state. The potential energy due to capillarity of a single drop can be estimated by assuming a drop can be represented by one half of a wavelength of a long cylindrical body of liquid when displaced from its equilibrium position, as shown in Fig. 5. The drop is assumed to be free of external constraints.

The position vector r of the surface of the drop can be expressed in a series form as

$$r = a + \sum a_n \cos n \theta \cos kz \quad (2)$$

where the coefficients a_n are functions of time of the assumed form $a_n \cos (pt + e)$ and where the summation starts at $n = 1$.

For such a configuration, Rayleigh (Ref. 4, p. 353) has shown that the surface energy E_p , due to capillarity, can be expressed using small perturbation

theory as

$$E_p = \frac{1}{4} \pi \sigma \frac{(k^2 a^2 + n^2 - 1)}{a} a_n^2 \quad (3)$$

where the wave number k is given as

$$k = \frac{2\pi}{\lambda_z} \quad (4)$$

The wavelength λ_z is related to the drop height by

$$h = \frac{\lambda_z}{2} \quad (5)$$

as shown in Fig. 5. Therefore,

$$k = \frac{\pi}{h} \quad (6)$$

The height h is assumed to be equal to the equilibrium height of a stationary drop resting on a surface with a contact angle of 180° (nonwetted surface). The values of h (Ref. 8) are given in table I as a function of the volume of the drop or its radius.

It is postulated that the drop will vibrate in the mode where the potential energy is near a minimum. Thus, setting

$$\left(\frac{\partial E_p}{\partial V} \right)_r = 0 \quad (7)$$

and noting that the equilibrium radius a is the only parameter assumed to depend on volume, yields

$$a^2 = \frac{n^2 - 1}{k^2} \quad (8)$$

For a given k and n , eq. (8) describes the drop radius associated with a state of minimum potential energy. This is easily shown by taking the second derivative of eq. (3) and substituting in eq. (8) to yield

$$\frac{1}{\frac{1}{4} \pi \sigma a_n^2} \frac{\partial^2 E_p}{\partial V^2} = \frac{2(n^2 - 1)}{a^3} \left(\frac{\partial a}{\partial V} \right)^2 \quad (9)$$

Since $(n^2 - 1) > 0$ and $a > 0$, it follows that

$$\frac{\partial^2 E_p}{\partial V^2} > 0 \quad (8)$$

Therefore, eq. (8) implies a state of minimum potential energy.

Using eq. (6), eq. (8) can be rewritten as

$$n^2 = 1 + \frac{2\pi^2}{h^2} \quad (9)$$

Substituting the equilibrium height from table I into eq. (9) yields

$$n^2 = 1 + 10.67 a^{*6/5} \quad 0.58 < a^* < 5.16 \quad (10)$$

$$n^2 = 1 + 2.88 a^{*2} \quad a^* > 5.16 \quad (11)$$

where the equilibrium radius a has been replaced

by its dimensionless form designated by the starred quantity. For all practical cases, the dimensionless volume of the drop is greater than 0.8 (for example - for water $V^* \approx 0.8$ is equivalent to $V = 0.0126 \text{ cm}^3$); therefore, equations are given for n only in the large drop and extended drop domains. As seen in eqs. (10) and (11), the number of nodes increase with increased radius a^* or the volume of the drop V^* .

This does not mean that placing an arbitrarily large drop in Leidenfrost boiling will cause vibrations. The volume may have to decrease to attain the proper size for modal oscillation. Also, any extraneous drop motion resulting from placing the liquid on the hot plate must in general die out before vibration can begin.

Eqs. (10) and (11) can also be written in terms of drop volume as

$$n^2 = 1 + 6.14 V^{*1/2} \quad 0.8 \leq V^* < 155 \quad (12)$$

$$n^2 = 1 + 0.5 V^* \quad V^* \geq 155 \quad (13)$$

In this form the equations are more useful experimentally since they prescribe the volume of liquid to be placed on the hot plate to obtain a given number of nodes. Eqs. (10) and (11), however, are more useful for correlations, since the radius can be read directly from the films.

Eqs. (10) and (11) are compared to the data in Fig. 6. As seen in Fig. 6, the theory follows the general trend of the data and bounds the data from below. An empirical correction factor will now be introduced to allow the theory to bound the data from above.

The values of the drop height h in table I have been found to give reasonable estimates for stationary (nonvibrating) drops in Leidenfrost film boiling. However, movie sequences of the drop in the vibration state indicate that h is sometime as large as twice the equilibrium height h . If the height h in eq. (9) is multiplied by a correction factor, C_f , then eqs. (10) and (11) become

$$n^2 = 1 + 10.67 \frac{a^{*6/5}}{C_f^2} \quad 0.8 \leq V^* < 155 \quad (14)$$

$$n^2 = 1 + 2.88 \frac{a^{*2}}{C_f^2} \quad V^* \geq 155 \quad (15)$$

Eqs. (14) and (15) can be made to bracket the data reasonably well. A value of C_f equal to 1.5 seems to work for the intermediate drops with 4 to 7 nodes. For small number of nodes like 2 or 3, C_f equals 2 is a good approximation where large drop height changes occur during an oscillatory cycle. For the very large drops, with 8 to 10 nodes, the drops remain relatively flat; therefore, the original theory, eqs. (10) and (11), are still a good approximation.

In most cases, therefore, the drop can be assumed to vibrate in a mode that requires a minimum amount of energy. For example, the most probable number of nodes for a 1 cm^3 drop of water ($a = 0.9 \text{ cm}$, $a^* = 3.57$) with $C_f = 1.5$ is 5 since all other

modes of vibration require more energy to excite them. More nodes will appear on larger drops while smaller drops will have fewer nodes on the average.

VIBRATIONAL FREQUENCIES

As mentioned in the Introduction, the natural frequencies of the drop were correlated in Ref. 3 using the equation

$$f = \frac{1}{2\pi} \left[(n^3 - n) \frac{\sigma}{\rho_L a^3} \right]^{1/2} \quad (16)$$

Eq. (16) was derived assuming the drop to be incompressible with the radius expanded in terms of $\cos n\theta$. Also the deviation from the equilibrium position was assumed small. The derivation paralleled Rayleigh's analysis for a spherical drop (Ref. 4, pp. 372-375) which expanded the drop radius in terms of Legendre's functions P_n .

In Ref. 3, the data for the natural frequencies were presented in terms of drop volume with the radius a in eq. (16) estimated from numerical solution to the Laplace capillary equation, eq. (1). In this paper, these data were reanalyzed using the equations given in table I. In addition to those data, the 9 and 10 mode water drop shown in Fig. 4, and some additional lower mode water drop data plus a 3 and 5 mode nitrogen drop were analyzed using the measured drop radius. As seen in Fig. 7, the measured frequencies of vibration for a given number of nodes agree quite well with the theoretical predicted frequencies from eq. (16).

Because of the dishing of the hot plate, the actual drop volume (mass) is larger than that estimated from the measured radius of the drop, using the equations of table I. If the drop were truly stationary and more pancake shaped, as indicated in table I, the effective drop radius to be used in eq. (16) would be larger. Consequently, the theory predicts slightly higher frequencies than observed, as seen by the data in Fig. 7.

INCIPIENT TEMPERATURE

As discussed earlier, large drop radii are required for vibrations with large number of nodes. Unfortunately, in Leidenfrost film boiling when the drop radius becomes somewhat larger than the critical wavelength from hydrodynamic stability theory, the vapor breaks through the center of the drop (bubble break through) and regular vibrations in distinct nodes are no longer possible.

The critical wavelength from hydrodynamic stability theory is given as (Ref. 11)

$$\lambda_c = 2\pi \left[\frac{g_c \sigma}{g(\rho_L - \rho_v)} \right]^{0.5} \quad (17)$$

As seen in eq. (17), a fluid with large surface tension σ will have a large λ_c ; therefore, a larger drop radius is possible without bubble break through. Water has a relatively large surface tension compared to other common fluids such as Ethanol, Benzene, Carbon tetrachloride or liquid nitrogen; therefore, it is the most convenient fluid to use in this study.

For water, an incipient temperature exists below

which free modal vibrations do not occur. This incipient plate temperature for water is approximately 412° C on stainless steel and 360° C on aluminum, or about 200° C above the Leidenfrost temperature. In this "quiescent" film boiling region, any vibration induced in the drop quickly damps out. For the other mentioned common fluids, modal vibrations can occur as long as the plate temperature is greater than the Leidenfrost temperature. However, because of the relatively low value of the critical wavelength, λ_c , for these fluids only the lowest nodal pattern, that of two node vibrations, were observed on a flat surface before vapor break through occurred. Water, on the other hand, could undergo all modal patterns from 2 to 6 nodes.

A reasonable explanation for this incipient temperature involves wetting which occurs between the drop and the surface. Nominally, in Leidenfrost boiling a thin vapor layer is assumed to exist between the drop and the heating surface. Bradfield's photographic study (Ref. 12), however, shows that thin spikes of liquid can reach down and touch the surface. These spikes can be expected to retard the free vibration of the drop, especially for water with its large surface tension coefficient. An oscillating drop has an extended surface which increases its potential for touching. Also, the vibrating drop creates a change in heat transfer at the surface which causes the surface of the water to be much closer to the metal surface.

When using a hypodermic syringe to form water drops, a cluster of small steam bubbles was always observed whenever water was placed on the hot plate; however, no vapor bubbles were seen for the other fluids. Therefore, water has a greater tendency to thermally interact with the hot surface (most likely because of its larger latent heat of vaporization) and thereby increase its frictional damping. The amount of this interaction was reduced by replacing the original stainless steel plate by an aluminum plate. This single change reduced the incipient vibration plate temperature by 50°. This change in the incipient temperature is most likely related to the change in the thermal properties (thermal conductivity, density, and specific heat) of the hot plate. A similar decrease in the Leidenfrost temperature was observed by replacing a stainless steel surface by an aluminum surface (Ref. 13 and 14). This lower plate temperature was still approximately 150° C above the Leidenfrost temperature. Thus, the thermal properties of the plate affect the viscous damping of the drop.

Yao and Henry (Ref. 15) performed a study of the fraction of a metal surface wetted by liquid spikes which occurs for a thin liquid in pool film boiling. They showed that the fraction of surface wetting falls off exponentially with increasing plate temperature. Compared to film boiling just above Leidenfrost temperature, as the plate temperature reaches the thermodynamic critical temperature, the percentage of surface wetting decreases by about two orders of magnitude. Therefore, the frictional retarding force on the water drop can be expected to decrease significantly as the plate temperature approaches the thermodynamic critical temperature. For water, the thermodynamic critical temperature is 374° C, which is in close agreement with the incipient vibration temperature of water for drops on an aluminum hot plate.

Because of the lower surface tension and perhaps the lower latent heat, other fluids such as Benzene, ethanol, etc. do not have an incipient temperature. Apparently, the surface interaction is too weak to prevent the modal oscillations of the liquid drops.

Circumferential Wave Propagation. Another phenomenon, which appears much less frequently than the vibrations, is the generation of a traveling wave around the circumference of the drop, as shown in Figs. 8 and 9. The wavelength observed corresponded to various exact integer divisions of the circumference; that is,

$$\lambda_R = \frac{2\pi a}{n} \quad (18)$$

Wave propagation was observed to occur for 2 to 6 nodes. We believe higher nodes are possible if bubble break through the random drop motion could be eliminated.

At first, this effect was mistaken as a general rotation of the drop. However, small particle tracers in the drop were observed to remain in the same location. They followed a path roughly similar to that shown in Fig. 10 for a wave propagating along a flat surface.

The wave phenomena for water was also observed for plate temperatures below the critical incipient temperature as well as above it, which is different from the radially vibrating drop.

Mechanism. As yet we still do not understand how the vibrations begin and how they are supported. The driving force might originate from the drop periodically touching the hot plate and receiving a burst of energy. This transfer of energy could occur through the small spikes discussed in conjunction with Refs. 12 and 15. Edge flutter resulting from the vapor jetting from beneath the drop could produce vertical vibrations which in turn could excite the observed radial vibration (see Ref. 16 and associated film supplement which show the uneven nature of the vapor flow beneath the drop). Uneven droplet surface temperature distributions or the large internal convection currents might also initiate the vibrations. The latter is a strong candidate for the mechanism which produces wave motion. However, at the present time, the cause of these vibrations and wave motions remains unresolved.

FORCED VIBRATIONS

Apparatus. To further our understanding of drop vibrations, a hot plate was attached to a shaker table which could be vibrated at various frequencies and amplitudes. This plate is similar to the previous plate used in the free vibration study except for four added bolt holes used to tie the plate onto the shaker table, see Figs. 3 and 11. Electrical heater coils were brazed to the bottom of the hot plate. During operation, cooling air was passed through the hold down bolt and beneath the hot plate to prevent heat transfer to the shaker table, as shown in Fig. 11.

Just as with the free vibration experiment, liquid was applied to the hot plate and the camera was used to record the motion whenever the drop began to oscillate.

Modal States. Modal states similar to those seen in free vibration experiments were observed in the forced vibration experiments. Fig. 12 shows a forced vibrational state for the third and fourth oscillatory modes. In general, the amplitude of the oscillation was considerably larger than for the free vibration state; compare with Figs. 1 and 2. Also, symmetry of the nodal pattern was often lacking as in the fourth modal state of Fig. 12.

As the amplitude of the shaker plate was increased the elongation of the nodes was noticeably increased. Fig. 12 depicts a large amplitude vibration of a 3 node drop. Further increases in the oscillation amplitude of the shaker table caused the drop to split apart. This is true for all modal shapes.

Vibrational Frequencies. Whenever the drop was observed to vibrate under forced conditions, the frequency of oscillation was found to be nearly equal to the driving frequency of the shaker table. The natural frequency of the drop as calculated from eq. (16) is also shown in Fig. 13. The calculated natural frequencies are slightly less than the driving frequency. This is somewhat surprising, since in forced vibration the deviations from equilibrium are exceptionally large when compared to free vibrations. Recall, that the derivation for the natural frequency was based on a small perturbation theory.

Incipient Temperature. Water could be forced vibrated at any temperature greater than the Leidenfrost temperature. Apparently, the driving force was sufficiently large to overcome any damping by the surface on the drop. In fact, the photographic results shown in Fig. 12 were performed on a plate whose temperature was below the incipient vibration temperature.

Wave Motion. Wave motion similar to that shown in Figs. 9 and 10 was also observed in the forced vibration experiments.

CONCLUSIONS

1. The vibrational energy contained in a drop is directly related to the radius of the drop and the number of nodes. In most cases, the drop will vibrate in a mode that requires a minimum of energy to excite.
2. There is good agreement between the theoretically calculated natural frequencies and the observed vibrational frequencies of drops.
3. An incipient temperature was found below which free oscillations of a water drop do not occur. This temperature could be approximated by the thermodynamic critical temperature. Below this temperature, wetting between the drop and its supporting plate exerts a sufficient retarding force to prevent oscillations.
4. Wave motion can exist along the circumference of the drop. The wavelength of this motion corresponds to various exact integer divisions of the circumference.
5. The natural modes of free vibration are exceptionally distinct and vigorous. Under forced vibration conditions, similar distinct vibrations are seen; however, the similarity of motion of all the modes are not observed. Rather, the drop maybe

somewhat distorted. If the amplitude of the oscillating force is sufficiently increased, the vibrating drop will split into fragments.

6. Small perturbation theory seems to reasonably predict the natural frequency of both free and for forced vibrating drops even for exceptionally large amplitudes of vibration.

REFERENCES

1. Leidenfrost, J. G., "De Aquae Communis Nonnullis Tractatus (A Tract about Some Qualities of Common Water, Duisburg on Rhine, 1756)," Univ. of Oklahoma, Norman, Oklahoma.
2. Holter, N. J. and Glasscock, W. R., "Vibrations of Evaporating Liquid Drops," J. Acoustical Soc. Am., 24, 682 (1952).
3. Schoessow, G. J. and Baumeister, K. J., "Liquid Drop Model Vibrations and Simulated Fission," Chem. Eng. Prog. Symp. Series, 62, 47 (1966).
4. Rayleigh, J. W., The Theory of Sound, Vol. II, pp. 343-375. Dover Publ., New York (1877).
5. Lamb, H., Hydrodynamics, pp. 473-479. Cambridge Univ. Press, London (1932).
6. Chiu, H. H., "Dynamics of Deformation of Liquid Drops," NYU-AA-68-45, New York Univ., N.Y. (1969).
7. Wolfson, B. T., "Oscillating Droplets," Research Rev., OAR, 6, 1 (1967).
8. Schoessow, G. J. and Baumeister, K. J., "Mass Diffusivity Effects on Droplets in Film Boiling," AIChE Symp. Series No. 118, 68, 156 (1972).
9. Tribus, M., Thermostatrics and Thermodynamics, Ch. 3, D. Van Nostrand, Princeton, N. J. (1961).
10. Den Hartog, J. P., Advanced Strength of Materials, McGraw-Hill, New York (1952).
11. Chang, Y. P., "Wave Theory of Heat Transfer in Film Boiling," J. Heat Transfer, 81, 1 (1957).
12. Bradfield, W. A., "Liquid-Solid Contact in Stable Film Boiling," Ind. Eng. Chem. Fundam., 5, 200 (1966).
13. Baumeister, K. J., Henry, R. E., and Simon, F. F., "Role of the Surface in the Measurement of the Leidenfrost Temperature," in Augmentation of Convection Heat and Mass Transfer, pp. 91-101, Am. Soc. Mech. Eng., New York (1970).
14. Baumeister, K. J. and Simon, F. F., "Leidenfrost Temperature - Its Correlation for Liquid Metals, Cryogenes, Hydrocarbons, and Water," J. Heat Transfer, 95, 166 (1973).
15. Yao, C. and Henry, R. E., "Hydrodynamic Instability Induced Liquid-Solid Contacts in Film Boiling," ASME Paper 76-S-792, 1976.
16. Hendricks, R. C. and Baumeister, K. J., "Heat Transfer and Levitation of a Sphere in Leidenfrost Boiling," NASA TN D-5694, and Film Supplement C-267, 1970.

TABLE 1. - EQUILIBRIUM DROP RADIUS AND HEIGHT FOR A DROP ON A FLAT SURFACE

WITH 180° CONTACT ANGLE

Dimensionless volume, $v^* = \frac{V}{L^3}$	Dimensionless radius, $a^* = \frac{a}{L}$	Dimensionless maximum radius, $a^* = \frac{a}{L}$	Dimensionless maximum height, $h^* = \frac{h}{L}$
Small spheroid $v^* < 0.8$	$a^* < 0.58$	$a^* = \left(\frac{3 v^*}{4\pi}\right)^{1/3}$	$h^* = 0.83 v^{*1/3}$
Large drop $0.8 < v^* < 155$	$0.58 < a^* < 5.16$	$a^* = \left(\frac{1.25 v^{*5/6}}{\pi}\right)^{1/2}$	$h^* = 0.8 v^{*1/6}$
Constant thickness $v^* > 155$	$a^* > 5.16$	$a^* = \left(\frac{0.54 v^*}{\pi}\right)^{1/2}$	$h^* = 1.85$

$$L = \left[\frac{\sigma g_c}{(\rho_L - \rho_V)g} \right]^{1/2}$$

$$v^* = \pi a^{*2} h^*$$



NODE NO. 2



3



4



5



6



7



8

Figure 1. - Modal states of drop in leidenfrost film boiling (ref. 3).



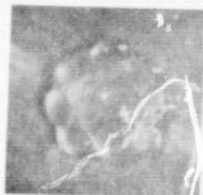
A



B



C



D



E

Figure 2. - Photographic history of one complete period of drop vibrating in the fourth mode (ref. 3).

NOTE - PLATE MATERIAL USED - ALUMINUM STAINLESS STEEL,
BRONZE, AND COPPER

HIGH-SPEED CAMERA
(90° TO TEST SECTION)

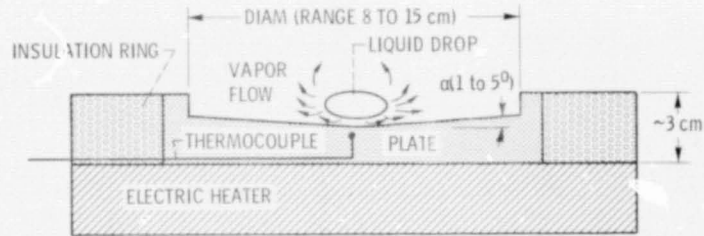
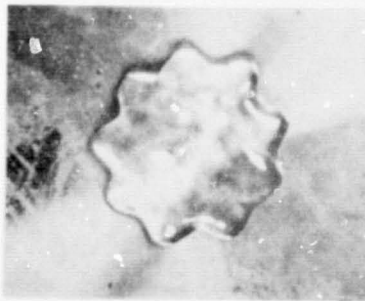
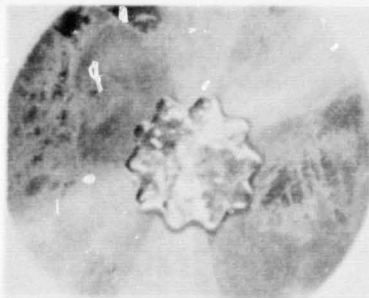


Figure 3. - Schematic of experimental set up.



(a) NINE NODE OSCILLATION.



(b) TEN NODE OSCILLATION.

Figure 4. - Oscillation of water drop in film boiling on 5° conical hot plate.

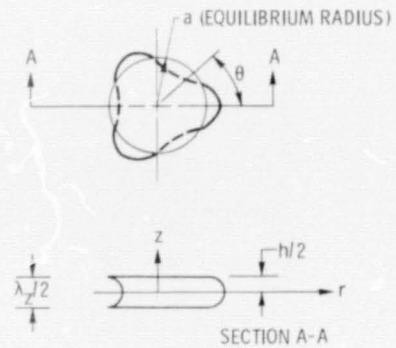


Figure 5. - Geometrical model.

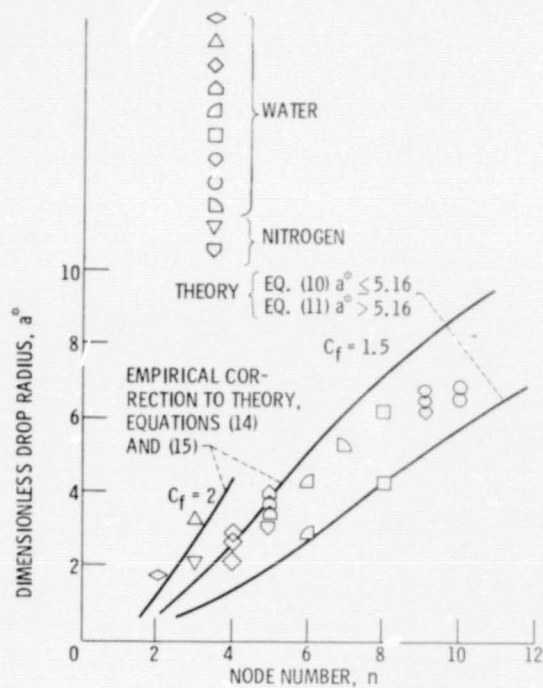


Figure 6. - Drop radius as a function of node number.

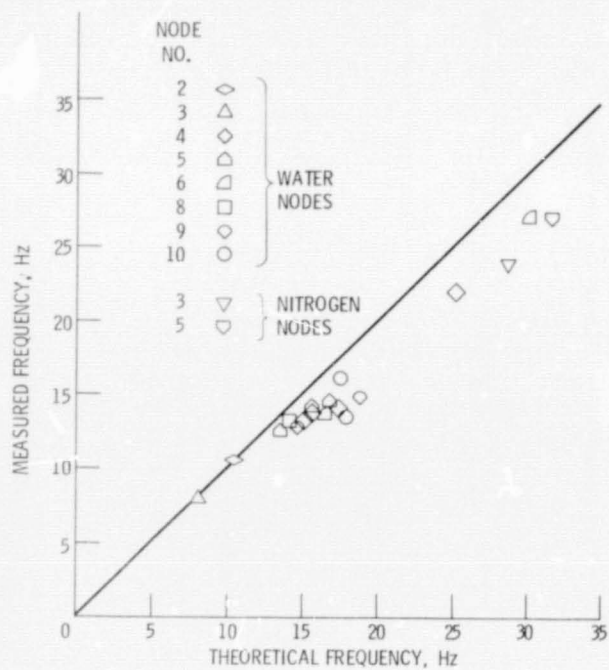


Figure 7. - Comparison of measured and theoretical natural frequencies.

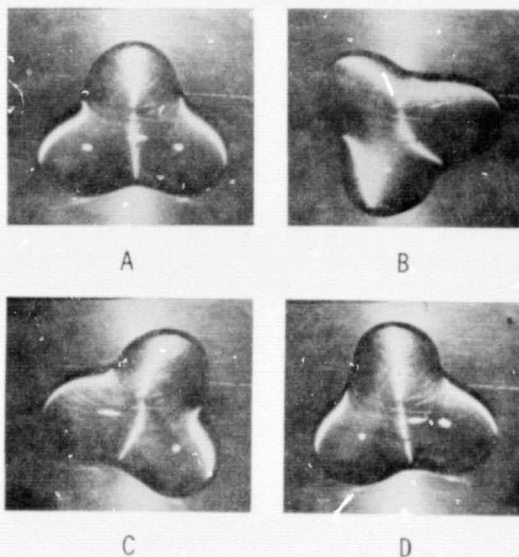


Figure 8. - Wave motion in 3 node state.

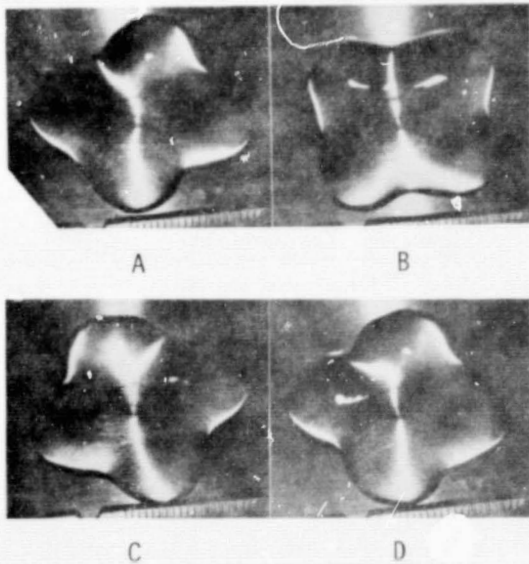


Figure 9. - Wave motion in 4 node state.

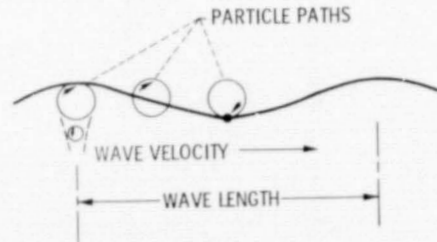


Figure 10. - Wave and tracer particle orbits.

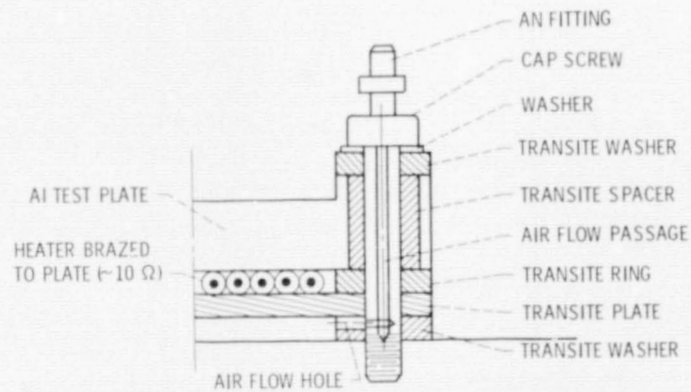
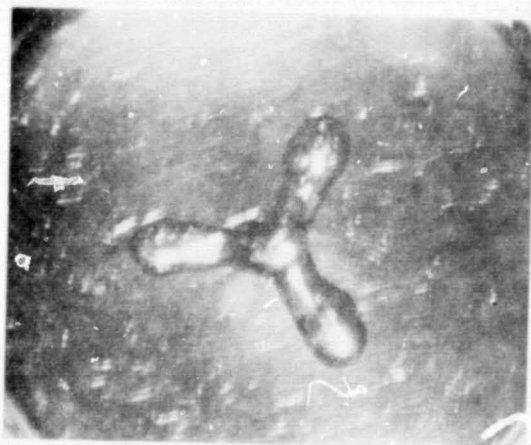


Figure 11. - Electrical heating and air cooling system for heated plate mounted on shaker table.

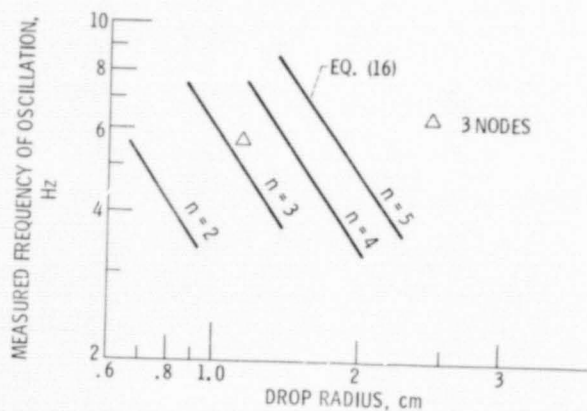


(a) THREE NODE OSCILLATION.

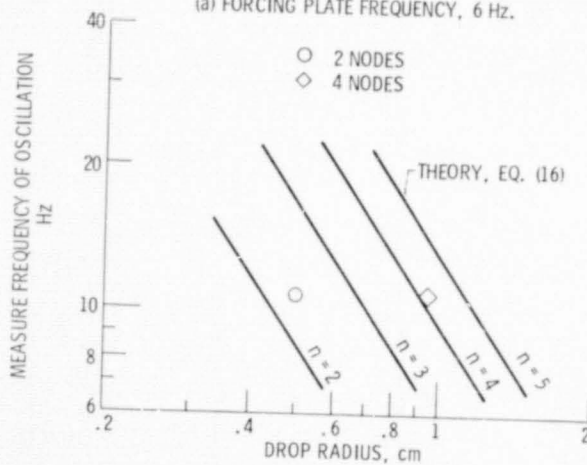


(b) FOUR NODE OSCILLATION.

Figure 12. - Oscillation of water drop in film boiling on plate attached to vibrating (12 cycles/sec) shaker table with table deflection of 0.05 cm.



(a) FORCING PLATE FREQUENCY, 6 Hz.



(b) FORCING PLATE FREQUENCY, 12 Hz.

Figure 13. - Comparison of measure drop oscillation to natural frequency for plate forcing frequencies of 6 and 12 with plate amplitude 0.05 cm for water ($\sigma = 58.8$ dynes/cm).